3. Earth pressures

Earth pressure is the most basic subject in foundation engineering.

In broad meaning: generic terminology of stresses observed in ground, except of pore water pressure,
similar to total and effective stresses in ground

In narrow meaning: vertical and horizontal pressures, wall pressures

Not only for the design of earth retaining structures, but also bearing capacity and slope stability analyses as well as ground deformation analysis need the knowledge of earth pressures.

how the earth pressures change in a ground;
how to estimate the earth pressures in various conditions.

3.1Earth pressures, earth pressure coefficients

In the conventional design of the retaining wall, **lateral earth pressure** is one of the most important input conditions. In the discussion of the earth pressure, if possible, we should better use effective stress separating pore water pressure from total stress.

In level ground, vertical earth pressures can be calculated from the equilibrium conditions as follows:

$$\sigma_v = \int_o^{z_0} \gamma(z)dz \quad (1)$$

$$\boxed{\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} &= 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} &= \gamma \end{aligned}}$$

$$\sigma'_v = \sigma_v - u(z_0) \quad (2)$$

Here σ_v and σ'_v are total and effective vertical stresses at the depth z_0 and $\gamma(z)$ and u(z) are bulk density and pore water pressure at the depth z. But horizontal stresses σ_h may not be determined from equilibrium conditions. ***(Why??)***

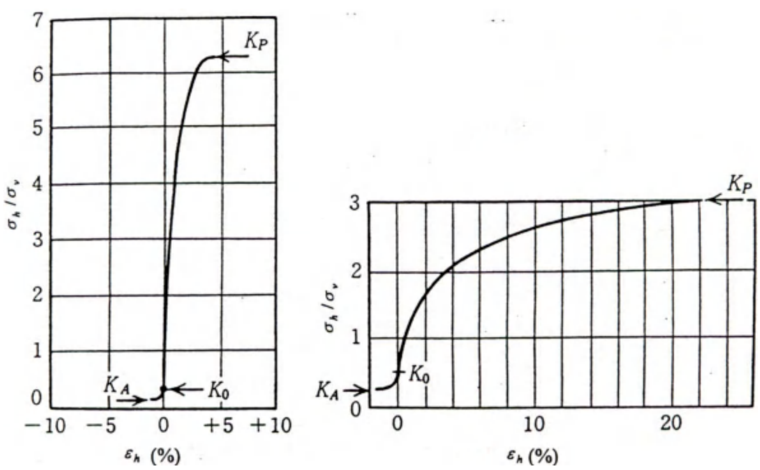
σ_h is statically indeterminate(不静定)

As the horizontal pressure acting on the wall varies according to the movement of the wall (i.e., strain as shown in **Fig.1**), additional condition on the strain of soil (compatibility condition) is necessary in order to determine the horizontal pressures. Normally horizontal pressure is described by using the earth pressure coefficient ***K***.

Effective stress : $p(=\sigma'_h) = K\sigma'_v$ (3) (有効)土圧

Total stress : $p_L(=\sigma_h) = K_L\sigma_v$ (4) 側圧

Some special cases or conditions to determine the horizontal stresses are earth pressure at rest, active pressure and passive pressure.



(a) dense sand (b) NC clay

Fig.1 Variation of horizontal stress ratio s_h/σ_v with horizontal strain ε_h

3.2 Earth pressure at rest

Initial condition is one of the crucial factors in the prediction of the behavior of the ground. Unless the proper initial conditions is specified, reliable results is hardly obtained from the analysis (ex, FEM, simple Winkler type spring model). **Earth pressure at rest (K_0 pressure:静止土圧)** is very important pressure, as it is normally adopted as the initial horizontal stress for level grounds.

From the definition of K_0 pressure, i.e.,ε_h=0, in isotropic elastic media K_0 pressure can be obtained from Hooke's law.

$$K_0 = \sigma'_h / \sigma'_v = \frac{\nu'}{1-\nu'} \qquad (5)$$

where ν' is Poisson 's ratio of the soil. Another elastic constant Young's modulus E has nothing to do with K_0. It should be noted that <u>K_0 value must be used for the effective stress not the total stress</u>.

If we introduce the earth pressure coefficient at rest in terms of the total stress:K_{L0}, the value is not constant in a same ground, but varies with depth as seen in **Fig.2**.

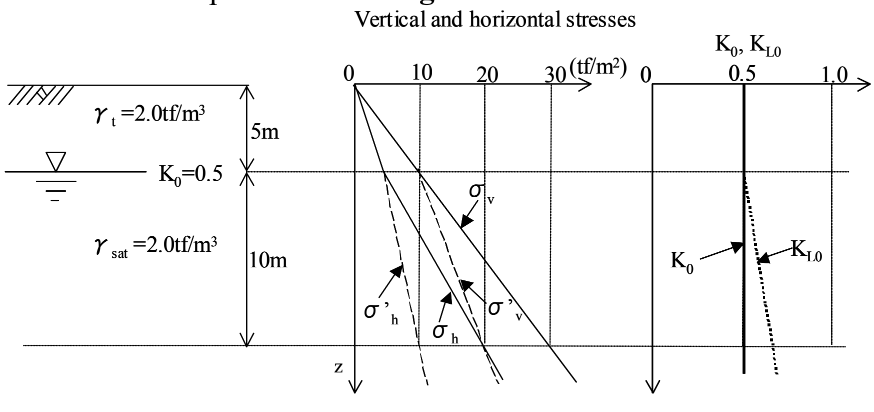


Fig.2 Variation of stresses and K_0 values with depth

Poisson's ratio ν' has to be given in order to evaluate K_0 value using Eq.(5). However, the determination of ν' is much more difficult than direct measurement of K_0. Hence some empirical equations are normally used for the determination.

1) Normally consolidated soils:

Jaky's equation using angle of internal friction φ' (**Fig.4**)

$$K_{0NC} = 1 - \sin\phi' \quad (6)$$

2) Overconsolidated soils:

For an identical soil, K_0 value varies with its stress history (OCR=σ'$_{max}$/σ'$_v$, unloading and reloading condition) as shown in **Fig.3**.

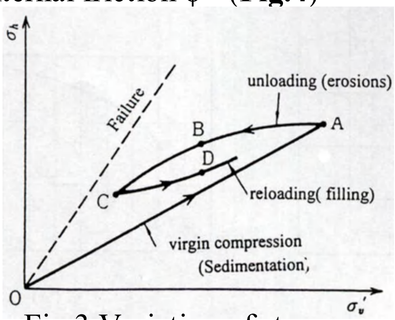
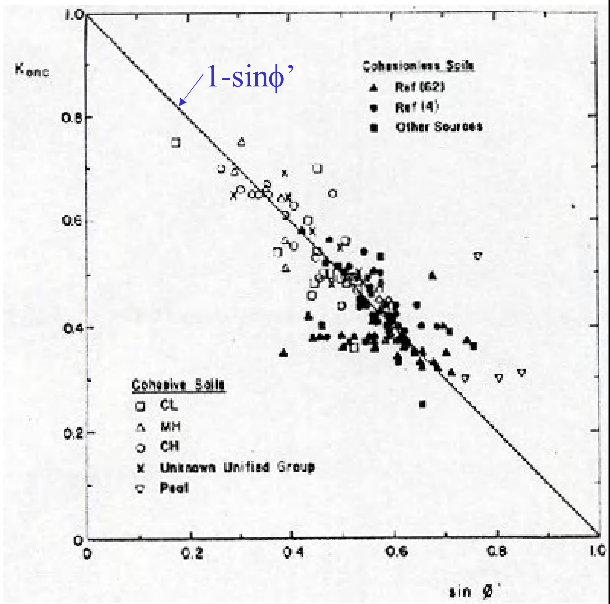


Fig.3 Variation of stress condition in K_0 condition

Fig.4
Observed relationship between K_{0NC} and sinφ' (Myane:1982)



Mayne's equation on K_0 of OC soils

Mayne[*] proposed the following equation for OC clay in both unloading and reloading conditions. ([*] Mayne, P. W. and Kulhawy, F. K.: K_0-OCR Relationships in Soils, Journal of Geotechnical Eng. Div. ASCE, Vol.108, No.GT6, pp.851-872, 1982.)

For unloading process:

$$K_{0U} = K_{0NC} \cdot OCR^{\alpha} \qquad (7)$$

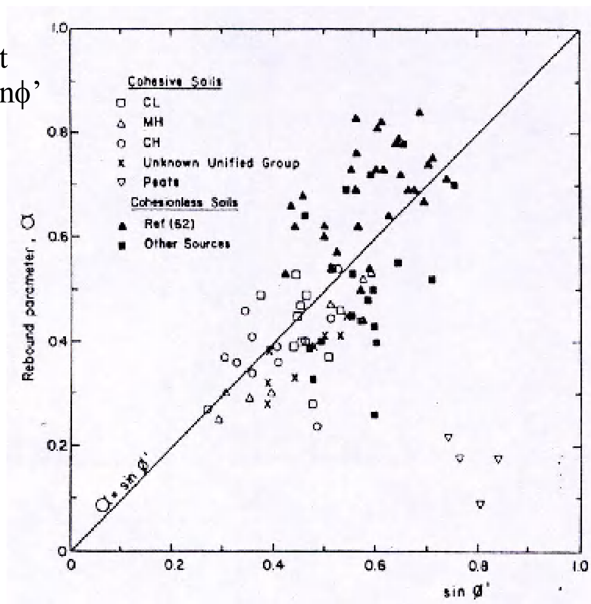
where α is rebound parameter. He found a good correlation between sin ϕ' and α and give the following equation. (**Fig.5**)

$$K_{0U} = K_{0NC} \cdot OCR^{\sin\phi'} \qquad (8)$$

For reloading process:

$$K_{0R} = K_{0NC}\left(\frac{OCR}{OCR_{\max}^{(1-\sin\phi')}}\right) + \frac{3}{4}K_{0NC}\left(1 - \frac{OCR}{OCR_{\max}}\right) \qquad (9)$$

Fig.5
Relationship between at rest rebound parameter α and sinϕ'
(Myane:1982)



Determination of K_0 using field tests

Pressure meter test

Dilatometer test

Hydro fracture test

3.3 Active and Passive pressures

Active and passive pressures are another very important design conditions. Stress conditions at failure make it possible to determine the both ultimate lateral pressures. The most common active and passive pressure equations are Rankine and Coulomb pressures.

(1) Rankine's stress at failure

Vertical stress in the level ground (**Fig.6**) at the depth z is given by

$$\sigma'_v = \gamma_t z + q \quad (10)$$

In a level ground σ'$_v$ and σ'$_v$ are both principle stresses (σ'$_1$ or σ'$_3$) and assumed to satisfy Mohr-Coulomb failure criteria at failure. (**Fig.7**)

active
passive
$$\pm(\sigma'_v - \sigma'_h) = (\sigma'_v + \sigma'_h)\sin\phi' + 2c'\cos\phi' \quad (11)$$

$$\pm(\sigma'_v - \sigma'_h) = (\sigma'_v + \sigma'_h)\sin\phi' + 2c'\cos\phi'$$

$$\tau_f = c' + \sigma'\tan\phi'$$

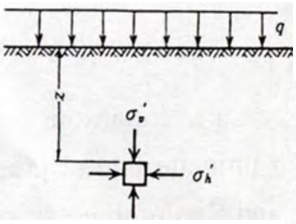
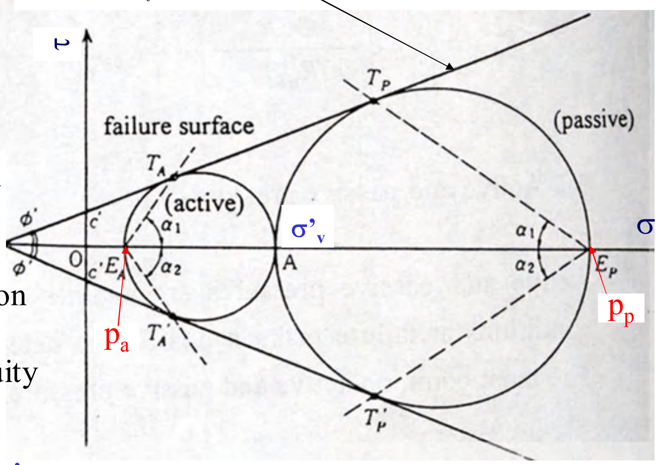


Fig.6 Stress in level ground



Uniform deformation is assumed.

no stress discontinuity

no rotation of σ'_1

lower bound analysis

slip line analysis

Fig.7 Mohr stress circles in failure conditions: level ground

And from Eq.(11), Equation of Rankine's pressures are derived.

For active pressure :

$$p_A = \sigma'_{hA} = (\gamma_t z + q)\tan^2\left(45^\circ - \frac{\phi'}{2}\right) - 2c'\tan\left(45^\circ - \frac{\phi'}{2}\right) \quad (12)$$

$$= (\gamma_t z + q)K_{AR} - 2c'\sqrt{K_{AR}}$$

For passive pressure :

$$p_P = \sigma'_{hP} = (\gamma_t z + q)\tan^2\left(45^\circ + \frac{\phi'}{2}\right) + 2c'\tan\left(45^\circ + \frac{\phi'}{2}\right) \quad (13)$$

$$= (\gamma_t z + q)K_{PR} + 2c'\sqrt{K_{PR}}$$

where K_{AR} and K_{PR} are coefficients of Rankine's pressure.

$$K_{AR} = \tan^2\left(45^\circ - \frac{\phi'}{2}\right) \quad (14)$$

$$K_{PR} = \tan^2\left(45^\circ + \frac{\phi'}{2}\right) \quad (15)$$

In the application of Rankine's pressure to the lateral pressure acting on the wall, smooth condition (i.e., τ=0)on the wall surface is theoretically required to secure the assumption that horizontal stress is a principle stress (σ_1 or σ_3). If considerable friction is expected on the wall, it should be taken into account especially for the passive pressure.

(2) Coulomb pressures

By using the Coulomb's earth pressure, the friction on the wall surface can be taken into account. Coulomb's earth pressures are given by the total force acting on the wall.

Coulomb's earth pressure theory is first application of **LEM** in geotechnical problem.

In the derivation of Coulomb pressure, straight slip plane, which forming a triangular wedge, is assumed.

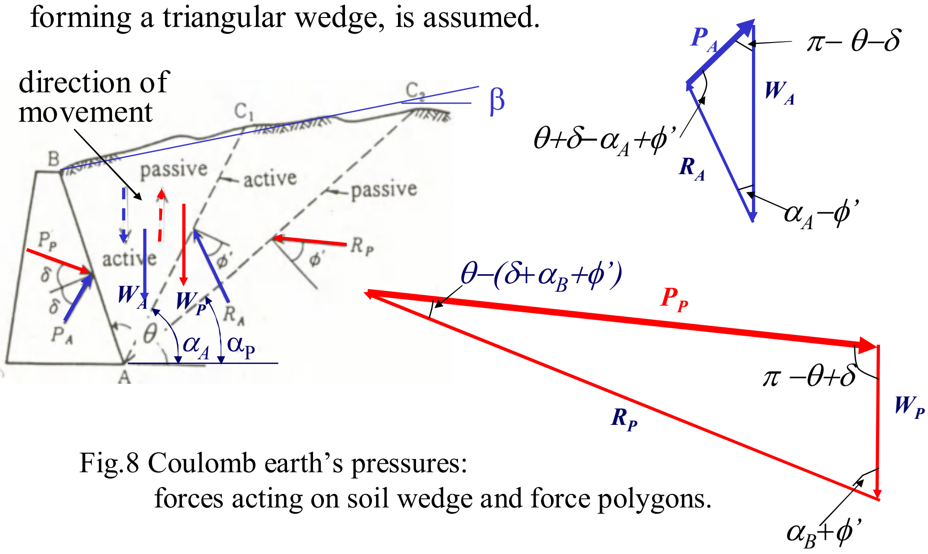


Fig.8 Coulomb earth's pressures:
forces acting on soil wedge and force polygons.

from force polygons

$$P_A = \frac{\sin(\alpha_A - \phi)}{\sin(\theta + \delta - \alpha_A + \phi)} W_A \qquad P_P = \frac{\sin(\alpha_P + \phi)}{\sin(\theta - \delta - \alpha_P - \phi)} W_P$$

$$W_A = f(\alpha_A, H, \beta, \theta) \qquad W_P = f(\alpha_P, H, \beta, \theta)$$

As H,β and θ are given (design) conditions, Coulomb's earth pressure (force) can be obtained

by maximization of P_A about α_A for active pressure: $\dfrac{\partial P_A}{\partial \alpha_A} = 0$

by minimization of P_P about α_P for passive pressure: $\dfrac{\partial P_B}{\partial \alpha_B} = 0$

For active: P_A

$$P_A = \frac{1}{2} \cdot \gamma_t \cdot H^2 \cdot K_{AC}$$

$$K_{AC} = \frac{\sin^2(\theta - \phi')}{\sin^2\theta \cdot \sin(\theta + \delta)} \left\{ 1 + \sqrt{\frac{\sin(\phi' + \delta) \cdot \sin(\phi' - \beta)}{\sin(\theta + \delta) \cdot \sin(\theta - \beta)}} \right\}^{-2} \qquad (16)$$

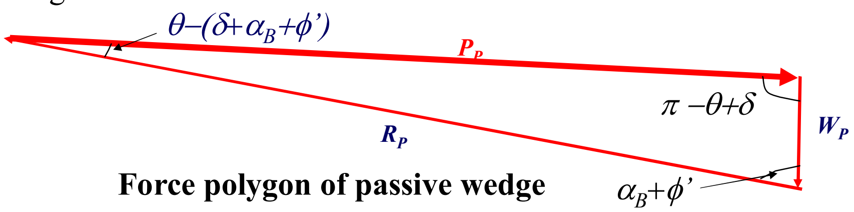
For passive : P_P

$$P_P = \frac{1}{2} \cdot \gamma_t \cdot H^2 \cdot K_{PC}$$

$$K_{PC} = \frac{\sin^2(\theta + \phi')}{\sin^2\theta \cdot \sin(\theta - \delta)} \left\{ 1 - \sqrt{\frac{\sin(\phi' + \delta) \cdot \sin(\phi' + \beta)}{\sin(\theta - \delta) \cdot \sin(\theta - \beta)}} \right\}^{-2} \qquad (17)$$

where K_{AC} and K_{PC} are coefficients of Coulomb's earth pressures, δ friction angle of the wall surface and θ inclination of wall and β slope angle of the back fill. **(Fig.8)**

In the derivation of Coulomb pressure, straight slip plane, which forming a wedge, is assumed. But the actual slip surface near the wall with frictional surface is curved (**Fig.9**). This discrepancy gives some error in the prediction of earth pressures, especially for the internal friction ϕ’. passive pressure with high wall surface friction δ and angle of



Force polygon of passive wedge

In case of vertical wall, θ=90°, maximum value of ϕ’ with full mobilization of wall friction, δ=ϕ’, is 45°. *over 45° no solution.*

ex) For ϕ’ and δ=40°, and θ=90°, β=0, K_{PC}=92.

(3) Wall pressures with curved failure surface

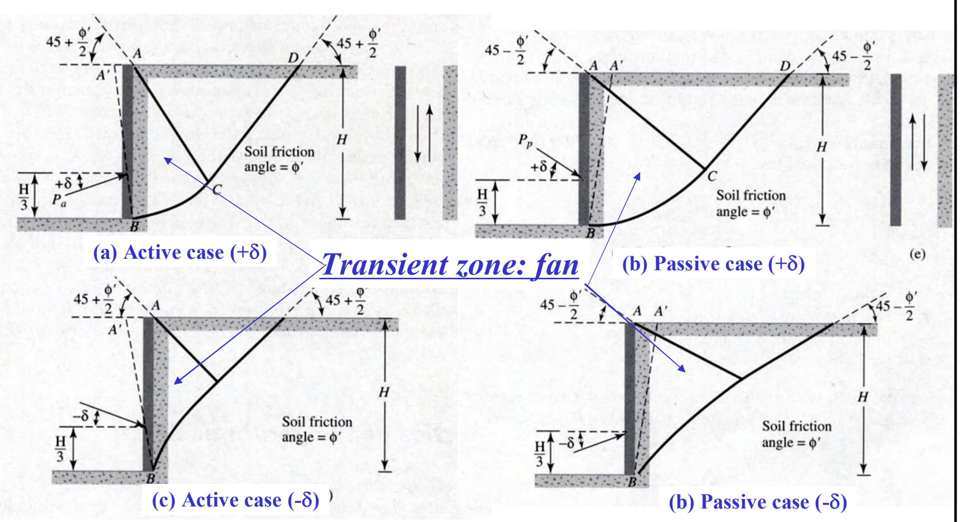
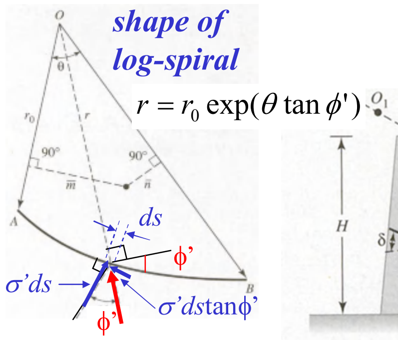
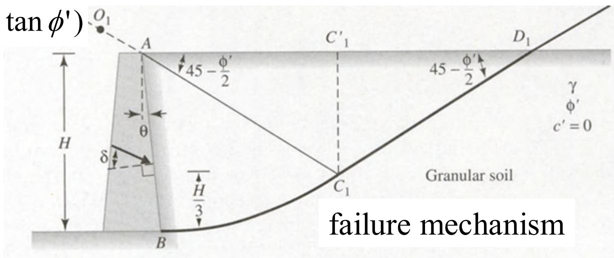


Fig.9 Effect of wall friction of wall surface

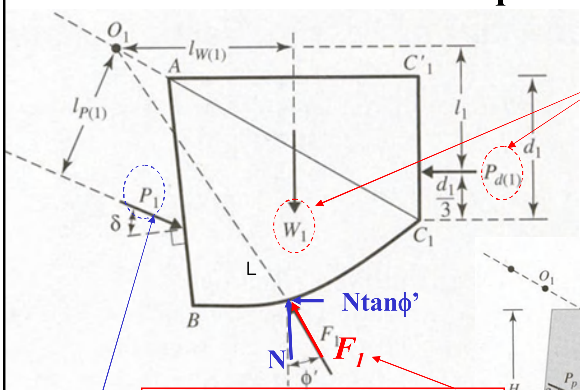
Fig.10 Passive earth pressure with curved failure surface log-spiral(ϕ'>0)

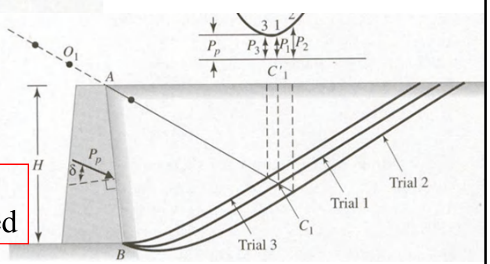




On every small portion of a log-spiral curve (*ds*), the direction of resultant forces caused by normal stress and frictional shear stress passes the center of log- spiral(O). Therefore in the log-spiral curve, it is not necessary to consider the resultant forces acting on the curve in the moment equilibrium on the point O. (*But not the case for cohesive force!!*)

Moment equilibrium





minimization of PP changing location of O.

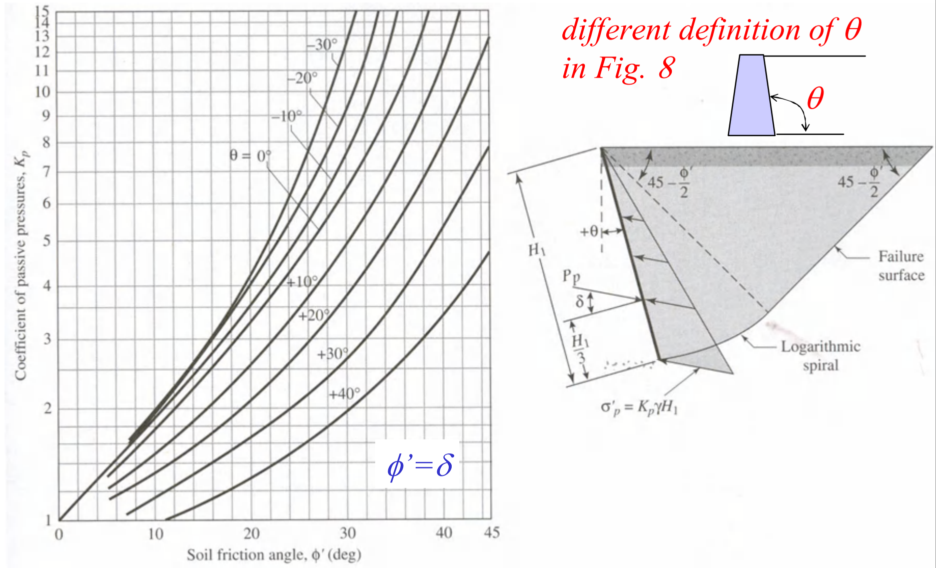


Fig.11 Caquot and Kerisel's solution for K_P

Effect of wall friction δ

$$K_p = R\, K_{p(\delta/\phi'=1)} \quad (18)$$

Rankine's Coloumb's pressures

Table 1 Caquot and Kerisel's Reduction Factor, R, for Passive Pressure Calculation

	δ/ϕ'							
ϕ'	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
10	0.978	0.962	0.946	0.929	0.912	0.898	0.881	0.864
15	0.961	0.934	0.907	0.881	0.854	0.830	0.803	0.775
20	0.939	0.901	0.862	0.824	0.787	0.752	0.716	0.678
25	0.912	0.860	0.808	0.759	0.711	0.666	0.620	0.574
30	0.878	0.811	0.746	0.686	0.627	0.574	0.520	0.467
35	0.836	0.752	0.674	0.603	0.536	0.475	0.417	0.362
40	0.783	0.682	0.592	0.512	0.439	0.375	0.316	0.262
45	0.718	0.600	0.500	0.414	0.339	0.276	0.221	0.174

Comparison of K_p obtained from various methods

	δ (º)	Coulomb (LEM:straight line)	Caquot.Kerisel (LEM:log-spiral)	Sokolovsky (Slip line method)
φ'=30º	0	3.0	3.0	3.0
	10	4.2	4.2	4.2
	20	6.1	5.4	5.4
	30	10.0	6.4	6.6
φ'=35º	0	3.7	3.7	3.7
	10	5.3	5.3	5.3
	20	8.3	7.5	7.3
	30	15.3	9.7	9.6
φ'=40º	0	4.6	4.6	4.6
	10	7.0	7.0	7.1
	20	11.9	10.4	9.7
	30	25.1	14.6	13.9
	40	92.3	17.5	18.2

3.4 Earth pressure in clay and sand

From Eq.s (11) to (17), effective stress and γ_t are used, which means these Eq.s correspond to earth pressure above the water table where the total stresses in the soil are the same as the effective stresses. In saturated clay layers, however, evaluation of excess pore water pressure due to shearing is so difficult that the failure criteria in terms of effective stress cannot be adopted in the design. Instead, the failure criteria in terms of total stress (ϕ_u=0; **Fig.12(a)**) is usually used for the clay by the assumption of undrained condition because the migration or seepage of water in clay is so slow compared with duration of construction.

$K_{AR}=K_{PR}=1$ obtained from Eq.s (14) &(15) with the condition of ϕ_u=0, and the undrained shear strength c_u derives the following equations from Eq.s (12) &(13) of Rankin's pressure.

for active: $$p_{LA}=\sigma_A=(\gamma_{sat}z+q)-2c_u \quad (19)$$

for passive: $$p_{Lp}=\sigma_P=(\gamma_{sat}z+q)+2c_u \quad (20)$$

On the other hand, fully drained condition ($\Delta u=0$) can be achieved in sand layers for normal construction speed and zero cohesion can be usually assumed under water in the sand (**Fig.12(b)**). **Fig.13** shows typical examples of horizontal earth pressure in both clay and sand layers. In clay layers with small c_u no much difference in the active and passive pressure, while in sand layers water pressure dominates in the lateral pressure in the active side.

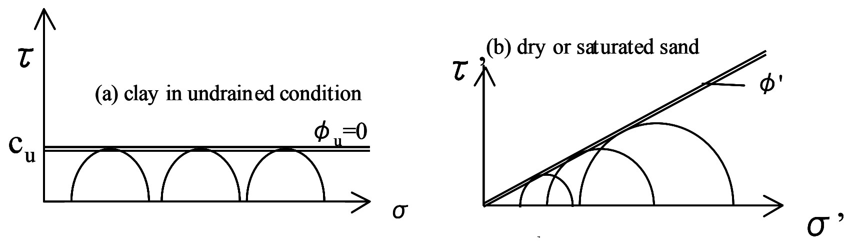


Fig.12 Failure criteria for clay and sand

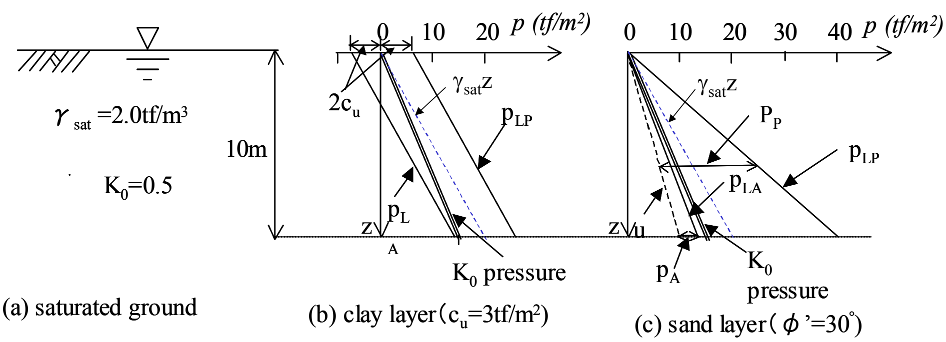


Fig. 13 Lateral earth pressures in clay and sand layers

3.4 Variation of earth pressure with wall movement

Earth pressure distributions along the flexible wall like sheet pile wall does not linearly increase like the distributions shown in **Fig.13**, but vary according to the deflection of the wall (see **Fig**. **14,15**). From many observations in various soil conditions (**Fig.16**), empirical distributions are used in the design mainly for the design of struts.

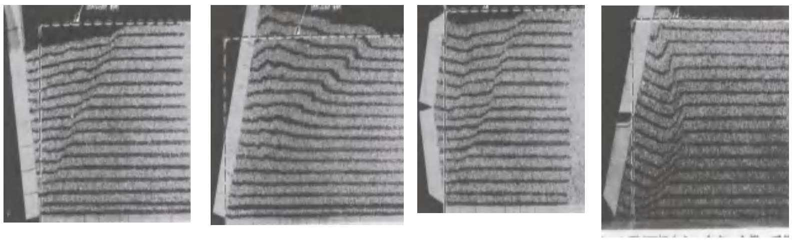


Fig.14 Effect of wall deflection on failure plane

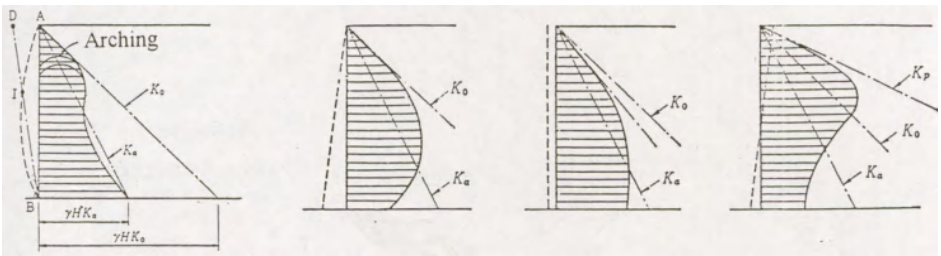


Fig.15 Lateral earth pressure distributions associated with wall movement

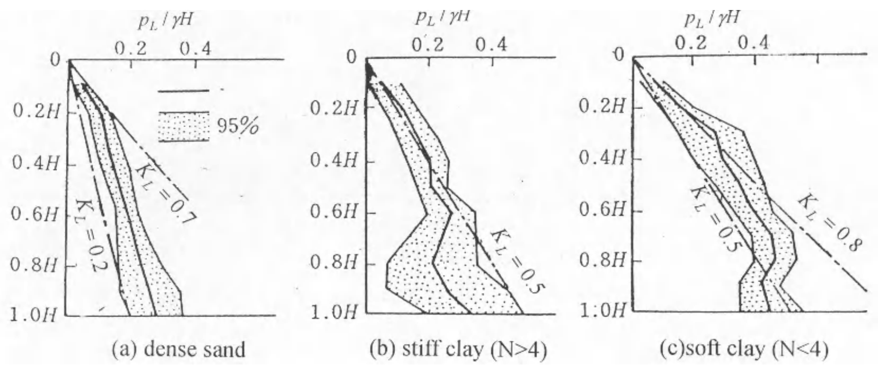
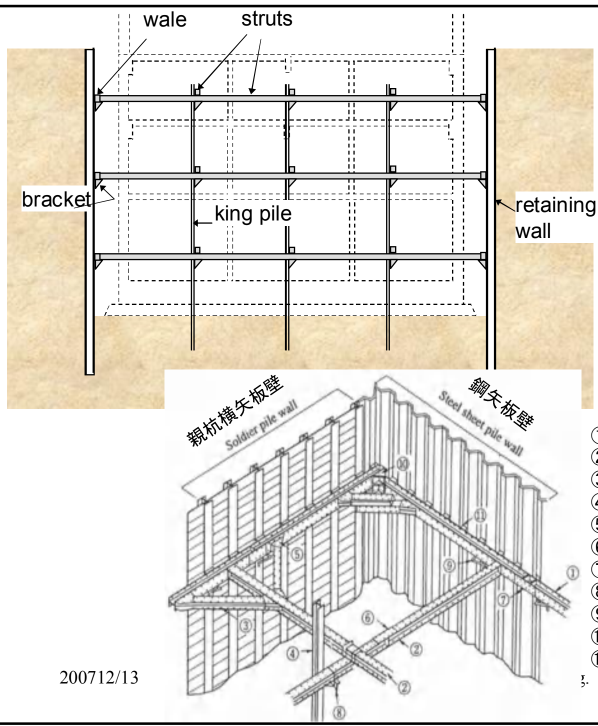


Fig.16 Lateral earth pressure distributions observed from previous observations

Earth retaining systems of strutted wall excavation

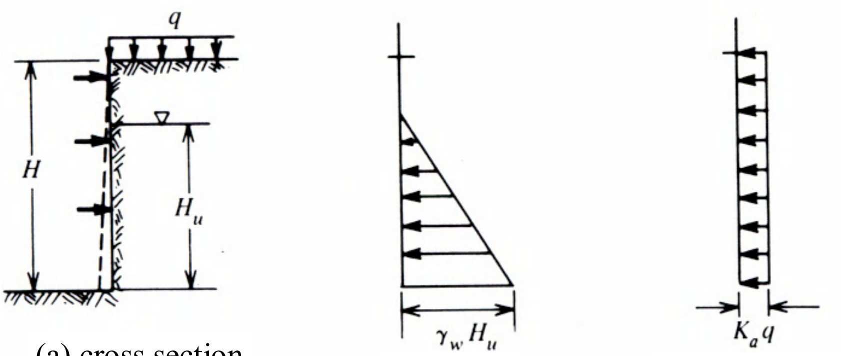


①Wale(腹起こし)
②Strut(切梁)
③Horizontal angle beam(火打ち)
④Post (King pile) (中間杭)
⑤Horizontal angle beam piece
⑥Strut cover piece
⑦Wale cover piece
⑧Post bracket
⑨Wale bracket
⑩Corner piece
⑪Concrete packing

Lateral pressure distribution for computation of strut(切梁) and tieback, wales(腹おこし) and ring stiffener in cofferdams(締切り):

water and surcharge

Foundation engineering Handbook 2nd ed.1991

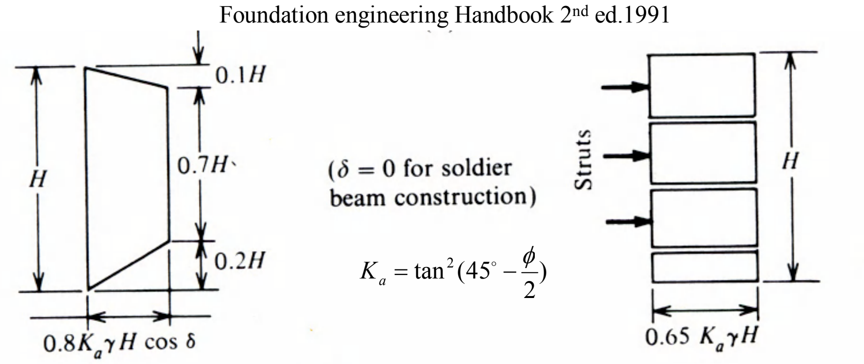


(a) cross section (b) Water (c) Surcharge

Lateral pressure distribution for computation of strut and tieback, wales and ring stiffener in cofferdams:

sand

Foundation engineering Handbook 2nd ed.1991

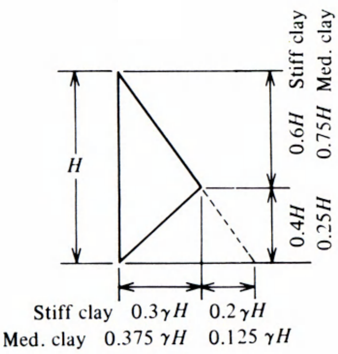


(d) sand(Tschebotarioff,1951) (e) sand (Peck,1969)

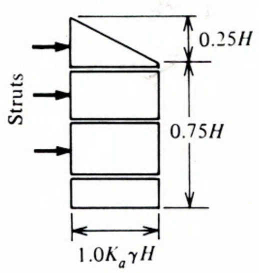
Lateral pressure distribution for computation of strut and tieback, wales and ring stiffener in cofferdams:
clay

Foundation engineering Handbook 2nd ed.1991

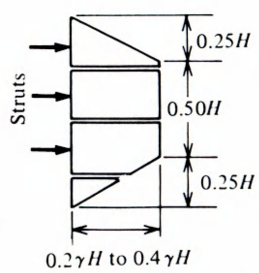
$K_a = 1 - m(2q_u/\gamma H)$ m: reduction factor depending on N= γH/c



(f) clay
(Tschebotarioff,1951)



(g) soft to medium clay

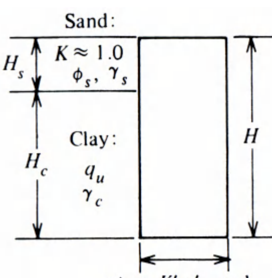


(h) stiff fissured clay

(Terzaghi and Peck,1967)

Lateral pressure distribution for computation of strut and tieback, wales and ring stiffener in cofferdams:
Others

Foundation engineering Handbook 2nd ed.1991



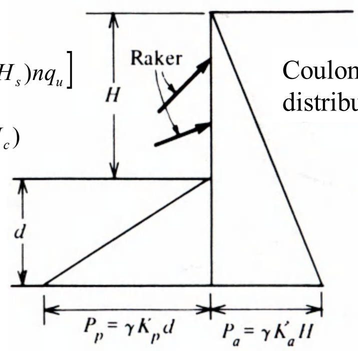
$$q'_u = \frac{1}{H}\left[\gamma_s H_s^2 K_s \tan\phi + (H - H_s) n q_u\right]$$

and $\gamma'_m = \frac{1}{H}(\gamma_s H_s + \gamma_c H_c)$

n: ratio of field shear strength of soil to laboratory value

$p'_m = K'_a \gamma'_m$ and
$K'_a = 1 - m(2q'_u/\gamma'_m H)$

(i) mixed soils



Coulomb distribution

(j) raker braced